

# PHASE TRANSITION AND HYBRID STAR IN A SU(2) CHIRAL SIGMA MODEL

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## Abstract

We use a modified SU(2) chiral sigma model to study nuclear matter at high density using mean field approach. We also study the phase transition of nuclear matter to quark matter in the interior of highly dense neutron stars. Stable solutions of Tolman-Oppenheimer-Volkoff equations representing hybrid stars are obtained with a maximum mass of  $1.69M_{\odot}$ , radii around 9.3 kms and a quark matter core constituting nearly 55-85% of the star radii.

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# 1 Introduction

It has been argued that nuclear matter undergoes a phase transition to quark matter at high densities and/or high temperatures. It is expected that the high temperature limit has interesting consequences in heavy ion collision and/or in cosmology, but high baryon density behavior is important to the study of neutron stars .

The quark structure of hadrons implies that at sufficiently large nuclear densities nuclear matter should convert into quark matter. The density at which transition occurs, is believed to be a few times nuclear matter density . The lattice calculations indicate that for nonzero quark masses the phase transition may be weak first order or second order<sup>1</sup>. Most of the model calculations find it to be first order. Thus for large enough mass of neutron star, its core may consists of quark matter. In addition, if the phase transition is first order, a part of the core may consists of mixed phase of quark and nuclear matter. Kapusta and others<sup>2,3</sup> have used a nonlinear Walecka model for the nuclear phase and a bag model for the quark phase . They have found that if the hadron-quark transition density  $n_B \geq 4n_0$  ( $n_0$  is the nuclear matter density  $=0.153\text{fm}^{-3}$ ), then it is quite unlikely that stable stars with quark interior exist. Ellis *et al.*<sup>3</sup> have also studied the possibility of a second order hadron-quark phase transition. In such cases, one needs to include an additional phenomenological parameter. A number of studies using different models have also been undertaken<sup>4-7</sup> showing a first order phase transition.

In all the above works,  $\mu_B$  (baryon chemical potential) is the only conserved charge and pressure remains constant in the mixed phase. Glendenning *et al.*<sup>8</sup> and Burgio *et al.*<sup>9</sup> have considered both  $\mu_B$  and  $\mu_E$  (electron chemical potential ) as conserved charges. This has the consequence that pressure varies continuously for all mixtures of the two components(hadron and quark) throughout the mixed phase.

In the present work, we have used a modified SU(2) chiral sigma model(MCH)<sup>10</sup> for hadronic matter since chiral model has been very successful, as such, in describing high density nuclear matter. The importance of chiral symmetry<sup>11</sup> in the study of nuclear matter was first emphasized by Lee and Wick<sup>12</sup> and has become over the years, one

of most useful tools to study high density nuclear matter at the microscopic level. The nonlinear terms in the chiral sigma model give rise to the three-body forces which become significant in the high density regime<sup>13</sup>. Further, the energy per nucleon at saturation needed the introduction of isoscalar field<sup>14</sup> in addition to the scalar field of pions<sup>15</sup>. We also include the interaction due to isospin triplet  $\rho$ -vector meson to describe the neutron rich-matter<sup>16</sup>.

The modified SU(2) chiral sigma model<sup>10</sup> considered by us includes two extra higher order scalar field interaction terms which ensures an appropriate incompressibility of symmetric nuclear matter at saturation density. Further, the equation of state(EOS) derived from this model is compatible with that inferred from recent heavy-ion collision data<sup>17</sup>.

In our work, we consider the baryon chemical potential  $\mu_B$  as the only conserved charge. Consequently pressure remains constant in the mixed phase region. A first order phase transition between beta stable nuclear matter and quark matter is indicated. Taking the existence of such a phase transition between nuclear matter and quark matter as a guide, we solve the Tolman-Oppenheimer-Volkoff(TOV) equations with appropriate nuclear matter<sup>10</sup> and quark matter<sup>18</sup> equations of state and find the hybrid stars to consist of a quark-matter core with the nuclear matter forming the crust.

This paper is organised as follows. In sec.2, we present the equation of state for nuclear matter. The quark matter equation of state is discussed in sec.3. In sec.4, we discuss the structure of hybrid star. We discuss and summarise our results in sec.5.

## 2 Nuclear matter Equation of State

The modified SU(2) chiral sigma model considered by us is described by the Lagrangian density<sup>10</sup>,

$$L = \frac{1}{2}(\partial_\mu \vec{\pi} \cdot \partial^\mu \vec{\pi} + \partial_\mu \sigma \partial^\mu \sigma) - \frac{1}{4}F_{\mu\nu}F_{\mu\nu} - \frac{\lambda}{4}(x^2 - x_0^2)^2 - \frac{\lambda B}{6m^2}(x^2 - x_0^2)^3 \\ - \frac{\lambda C}{8m^4}(x^2 - x_0^2)^4 - g_\sigma \bar{\psi}(\sigma + i\gamma_5 \vec{\tau} \cdot \vec{\pi})\psi + \bar{\psi}(i\gamma_\mu \partial^\mu - g_\omega \gamma_\mu \omega^\mu)\psi$$

$$+\frac{1}{2}g_\omega^2 x^2 \omega_\mu \omega^\mu - \frac{1}{4}G_{\mu\nu}.G^{\mu\nu} + \frac{1}{2}m_\rho^2 \vec{\rho}_\mu.\vec{\rho}^\mu - \frac{1}{2}g_\rho \bar{\psi}(\vec{\rho}_\mu.\vec{\tau}\gamma^\mu)\psi \quad (1)$$

In the above Lagrangian,  $F_{\mu\nu} \equiv \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$ ,  $G_{\mu\nu} \equiv \partial_\mu \rho_\nu - \partial_\nu \rho_\mu$  and  $x = (\vec{\pi}^2 + \sigma^2)^{1/2}$ ,  $\psi$  is the nucleon isospin doublet,  $\vec{\pi}$  is the pseudoscalar-isovector pion field,  $\sigma$  is the scalar field and  $\omega_\mu$ , is a dynamically generated isoscalar vector field, which couples to the conserved baryonic current  $j_\mu = \bar{\psi}\gamma_\mu\psi$ .  $\vec{\rho}_\mu$  is the isotriplet vector meson field with mass  $m_\rho$ . B and C are constant coefficients associated with the higher order self-interactions of the scalar field .

The masses of the nucleon, the scalar meson and the vector meson are respectively given by

$$m = g_\sigma x_0, \quad m_\sigma = \sqrt{2\lambda}x_0, \quad m_\omega = g_\omega x_0 \quad (2)$$

Here  $x_0$  is the vacuum expectation value of the  $\sigma$  field ,  $g_\omega$ ,  $g_\rho$  and  $g_\sigma$  are the coupling constants for the vector and scalar fields respectively and  $\lambda = (m_\sigma^2 - m_\pi^2)/(2f_\pi^2)$ , where  $m_\pi$  is the pion mass ,  $f_\pi$  is the pion decay coupling constant .

Mean-field approximation has been used extensively to obtain field theoretical equation of state for high density matter<sup>19</sup>. Using this approximation, the equation of motion for isoscalar vector field is

$$\omega_0 = \frac{n_B}{g_\omega x^2} \quad (3)$$

and the equation of motion for the scalar field in terms of  $y \equiv \frac{x}{x_0}$  is of the form<sup>10</sup>

$$(1 - y^2) - \frac{B}{m^2 C_\omega}(1 - y^2)^2 + \frac{C}{m^4 C_\omega^2}(1 - y^2)^3 + \frac{2C_\sigma C_\omega n_B^2}{m^2 y^4} - \frac{C_\sigma \gamma}{\pi^2} \int_0^{k_f} \frac{k^2 dk}{\sqrt{k^2 + m^{*2}}} = 0 \quad (4)$$

where  $m^* \equiv ym$  is the effective mass of the nucleon and the coupling constants are

$$C_\sigma \equiv \frac{g_\sigma^2}{m_\sigma^2}, \quad C_\omega \equiv \frac{g_\omega^2}{m_\omega^2} \quad (5)$$

The baryon number density  $n_B = n_p + n_n = \frac{\gamma k_f^3}{6\pi^2}$ , where  $k_f$  is the Fermi momentum and  $\gamma$  is the spin degeneracy factor which is equal to 4 and 2 for nuclear and neutron

matter respectively. The equation of motion for  $\rho$ , in the mean field approximation gives<sup>15</sup>

$$\rho_0^3 = (g_\rho/2m_\rho^2)(n_p - n_n) \quad (6)$$

At high densities the interior of neutron stars composed of asymmetric nuclear matter with an admixture of electrons. The concentrations of neutrons, protons and electrons can be determined using conditions of beta equilibrium and electrical charge neutrality<sup>20</sup>.

$$\mu_n = \mu_p + \mu_e ; n_p = n_e ,$$

(here  $\mu_i$  is the chemical potential of the particle species  $i$ ). In our analysis we find the ratio  $n_p/n_n$  to lie in the range 0.003-0.08 for  $n_B$  taking value from 0.02 fm<sup>-3</sup> to 0.25 fm<sup>-3</sup>. We have included the interaction due to isospin triplet  $\rho$ -meson in Eqn-1 for describing neutron-rich matter. The symmetric energy coefficient that follows from the semi-empirical nuclear mass formula is<sup>10,15</sup>

$$a_{sym} = \frac{C_\rho k_f^3}{12\pi^2} + \frac{k_f^2}{6\sqrt{k_f^2 + m^{*2}}} ,$$

where  $C_\rho \equiv g_\rho^2/m_\rho^2$ .

The diagonal components of the conserved total stress tensor corresponding to the Lagrangian(Eqn.1) together with the mean field equation of motion for the fermion field and a mean-field approximation for the meson fields is used to calculate the equation of state. The total energy density( $\epsilon$ ) and pressure( $P$ ), for the neutron rich nuclear matter in  $\beta$ -equilibrium is given by<sup>10</sup>

$$\begin{aligned} \epsilon &= \frac{m^2(1-y^2)^2}{8C_\sigma} - \frac{B}{12C_\omega C_\sigma}(1-y^2)^3 + \frac{C}{16m^2 C_\omega^2 C_\sigma}(1-y^2)^4 \\ &\quad + \frac{C_\omega n_B^2}{2y^2} + \frac{\gamma}{2\pi^2} \sum_{n,p,e} \int_0^{k_f} k^2 dk \sqrt{k^2 + m^{*2}} + \frac{1}{2} m_\rho^2 (\rho_0^3)^2, \\ P &= -\frac{m^2(1-y^2)^2}{8C_\sigma} + \frac{B}{12C_\omega C_\sigma}(1-y^2)^3 - \frac{C}{16m^2 C_\omega^2 C_\sigma}(1-y^2)^4 \\ &\quad + \frac{C_\omega n_B^2}{2y^2} + \frac{\gamma}{6\pi^2} \sum_{n,p,e} \int_0^{k_f} \frac{k^4 dk}{\sqrt{k^2 + m^{*2}}} + \frac{1}{2} m_\rho^2 (\rho_0^3)^2 \end{aligned} \quad (7)$$

The energy per nucleon is  $\frac{E}{A} = \frac{\epsilon}{n_B}$  and the chemical potential is  $\mu = (P + \epsilon)/n_B$ .

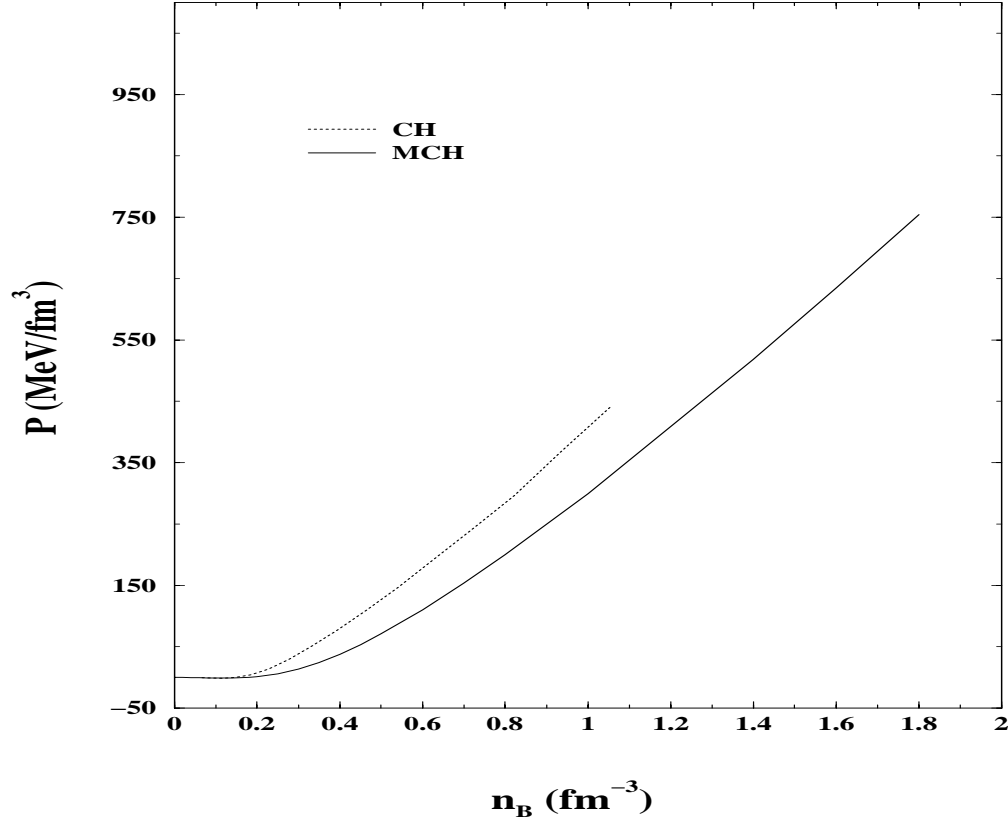


Figure 1: Pressure (P) vs. baryon number density( $n_B$ ) for nuclear matter

The values of five parameters  $C_\sigma, C_\omega, C_\rho$ , B and C occurring in the above equations are obtained by fitting with the saturation values of binding energy/nucleon (-16.3 MeV), the saturation density ( $0.153 \text{ fm}^{-3}$ ), the symmetric energy(32 MeV), the effective(Landau) mass ( $0.85M$ )<sup>21</sup>, and nuclear incompressibility ( $\sim 300 \text{ MeV}$ ), in accordance with recent heavy-ion collision data<sup>17</sup> are  $C_\omega = 1.999 \text{ fm}^2$ ,  $C_\sigma = 6.8157 \text{ fm}^2$ ,  $C_\rho = 4.661 \text{ fm}^2$ , B = -99.985 and C = -132.2456.

The pressure as a function of number density of nuclear matter is presented in Fig.[1]. The solid curve(MCH) corresponds to the model considered by us, whereas the dotted curve(CH) corresponding to the original chiral sigma model<sup>15</sup> is presented for comparison. The equation of state for the present model is found to be softer with respect to the original one.

### 3 Quark Matter Equation of State and Phase Transition.

Several authors<sup>22–25</sup> have studied the possible existence of quark matter in the core of neutron stars/pulsars. Densities of these stars are expected to be high enough to force the hadron constituents or nucleons to strongly overlap thereby yielding quark matter. Since the distance involved is small, perturbative Quantum Chromodynamics (QCD) is used to derive quark matter equation of state. We consider here the quark matter EOS which includes u, d and s quark degrees of freedom<sup>18,25</sup> in addition to electrons. We have taken the electron, up and down quark masses to be zero<sup>18</sup> and the strange quark mass is taken to be 180 MeV. In chemical equilibrium, one has  $\mu_d = \mu_s = \mu_u + \mu_e$  which can be written in terms of baryon and electric charge chemical potentials as<sup>5,6</sup>

$$\begin{aligned}\mu_u &= \frac{1}{3}\mu_B + \frac{2}{3}\mu_E, \\ \mu_d &= \frac{1}{3}\mu_B - \frac{1}{3}\mu_E, \\ \mu_s &= \frac{1}{3}\mu_B - \frac{1}{3}\mu_E \\ \mu_e &= -\mu_E\end{aligned}\tag{8}$$

The pressure contributed by the quarks is computed to order  $\alpha = \frac{g^2}{4\pi}$ , where  $g$  is the QCD coupling constant. The electron pressure is<sup>18</sup>

$$P_e = \frac{\mu_e^4}{12\pi^2},\tag{9}$$

and the pressure for quark flavor  $f$ , with  $f = u, d$  or  $s$  is<sup>3,18,25</sup>

$$\begin{aligned}P_f &= \frac{1}{4\pi^2}[\mu_f k_f(\mu_f^2 - 2.5m_f^2) + 1.5m_f^4 \ln(\frac{\mu_f + k_f}{m_f})] \\ &\quad - \frac{\alpha_s}{\pi^3}[\frac{3}{2}(\mu_f k_f - m_f^2 \ln(\frac{\mu_f + k_f}{m_f}))^2 - k_f^4]\end{aligned}\tag{10}$$

Where  $k_f = (\mu_f^2 - m_f^2)^{1/2}$  is the Fermi momentum. The total pressure, including the

bag constant  $B$  is given by<sup>18</sup>

$$P = P_e + \sum_f P_f - B \quad (11)$$

In the above equations  $\mu_B$  and  $\mu_E$  are only two independent chemical potentials.  $\mu_E$  is adjusted so that the matter is electrically neutral i.e  $\partial P / \partial \mu_B = 0$ . The baryon number density ( $n_B$ ) and the energy density ( $\epsilon$ ) for quark matter can be derived using the thermodynamic relations<sup>18</sup>

$$n_B = \partial P / \partial \mu_B \quad \text{and} \quad \epsilon = -P + \mu \frac{\partial P}{\partial \mu}.$$

Now we shall study the possible scenario of phase transition from nuclear matter to quark matter. Gibb's criteria is used to determine the phase boundary of the co-existence region between the nuclear and quark phase. The critical pressure and the critical chemical potentials are determined by the condition

$$P_{nm}(\mu_B) = P_{qm}(\mu_B) \quad (12)$$

We have taken typical values  $\alpha_s = 0.5, 0.6$  and the bag constant  $B = (155 \text{ MeV})^4$  and  $(150 \text{ MeV})^4$ , which are reasonable values to calculate pressure in the quark sector<sup>5</sup>.

We have plotted pressure versus chemical potential for beta stable nuclear matter and quark matter in Fig.[2]. The solid line is shown for the beta-stable nuclear matter with MCH parameter set. The dotted and dashed lines correspond to quark matter with  $\alpha_s = 0.5$  and  $\alpha_s = 0.6$  with bag pressure  $B = (155 \text{ MeV})^4$  respectively. We find that there exist phase transition points for nuclear matter at different pressures and chemical potentials. The transition point ( $P_{crit.}, \mu_{crit.}$ ) for  $\alpha_s = 0.6$  is (101 MeV/fm<sup>3</sup>, 1237 MeV) and that for  $\alpha_s = 0.5$  is (43 MeV/fm<sup>3</sup>, 1111 MeV). Fig.[3] shows the phase diagram with a different bag pressure  $B = (150 \text{ MeV})^4$ . Here the transition point ( $P_{crit.}, \mu_{crit.}$ ) for  $\alpha_s = 0.6$  is (27 MeV/fm<sup>3</sup>, 1064 MeV) and that for  $\alpha_s = 0.5$  is (7 MeV/fm<sup>3</sup>, 993 MeV). Thus, we find though the transition point increases with the increase of bag pressure and  $\alpha_s$ , with one of them remaining constant, the dependence on  $\alpha_s$  is more sensitive.

Now considering a typical transition point ( $P_{crit.}, \mu_{crit.}$ ) for  $B=(150 \text{ MeV})^4$ ,  $\alpha_s=0.5$ , at the critical pressure, the energy densities for the quark matter ( $\epsilon_{crit.}^{qm}$ ) and nuclear matter



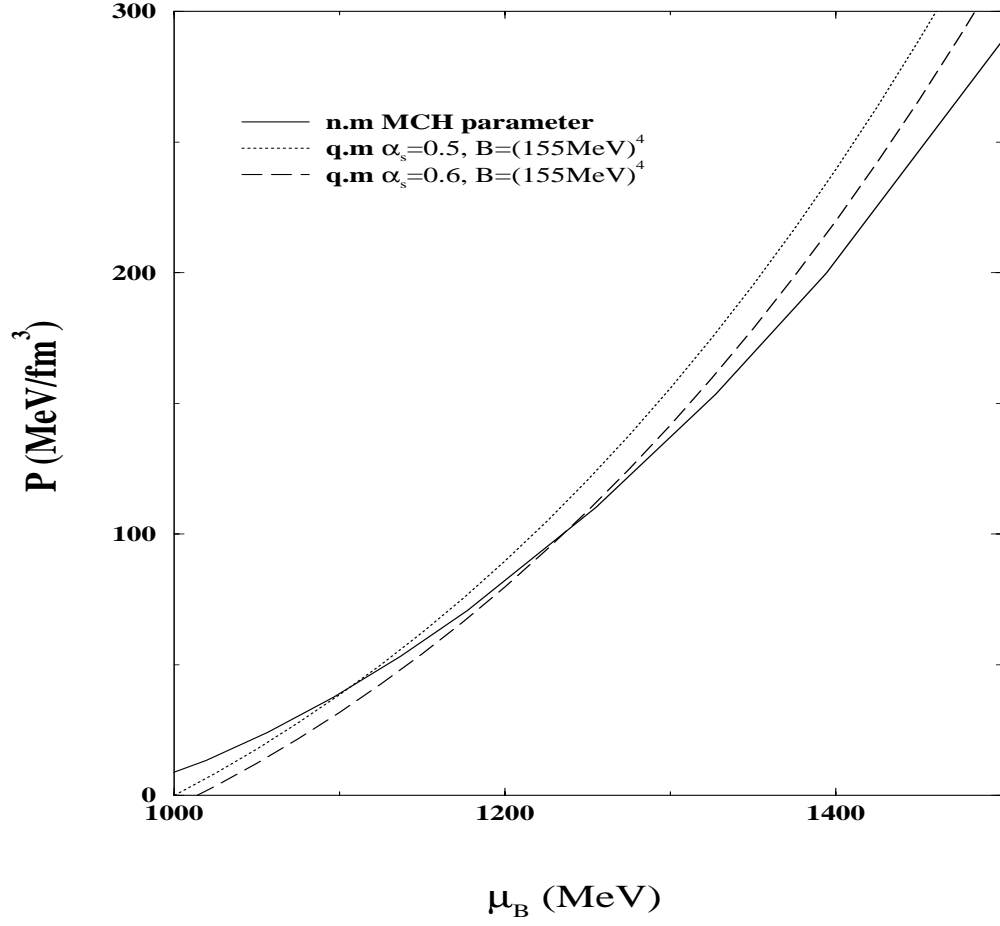


Figure 2: Pressure (P) vs. chemical potential for nuclear matter and quark matter with various  $\alpha_s$  at constant bag pressure (B)

( $\epsilon_{crit.}^{nm}$ ) sectors are found to be 387 MeV/fm<sup>3</sup> and 252 MeV/fm<sup>3</sup> respectively. The baryon number densities corresponding to the critical  $\mu_B$  in quark matter is  $n_B^{qm} = 0.37 fm^{-3}$  and that in nuclear matter is  $n_B^{nm} = 0.27 fm^{-3}$  which is reasonably an order of magnitude (about 1.8 to 2.5 times) higher than the nuclear matter density. The discontinuity in the number density as well as energy density indicates a first order phase transition. This phase transition from nuclear matter to quark matter obviously implies that the interior of neutron star consists of quark matter. We investigate this possibility further in the next section.

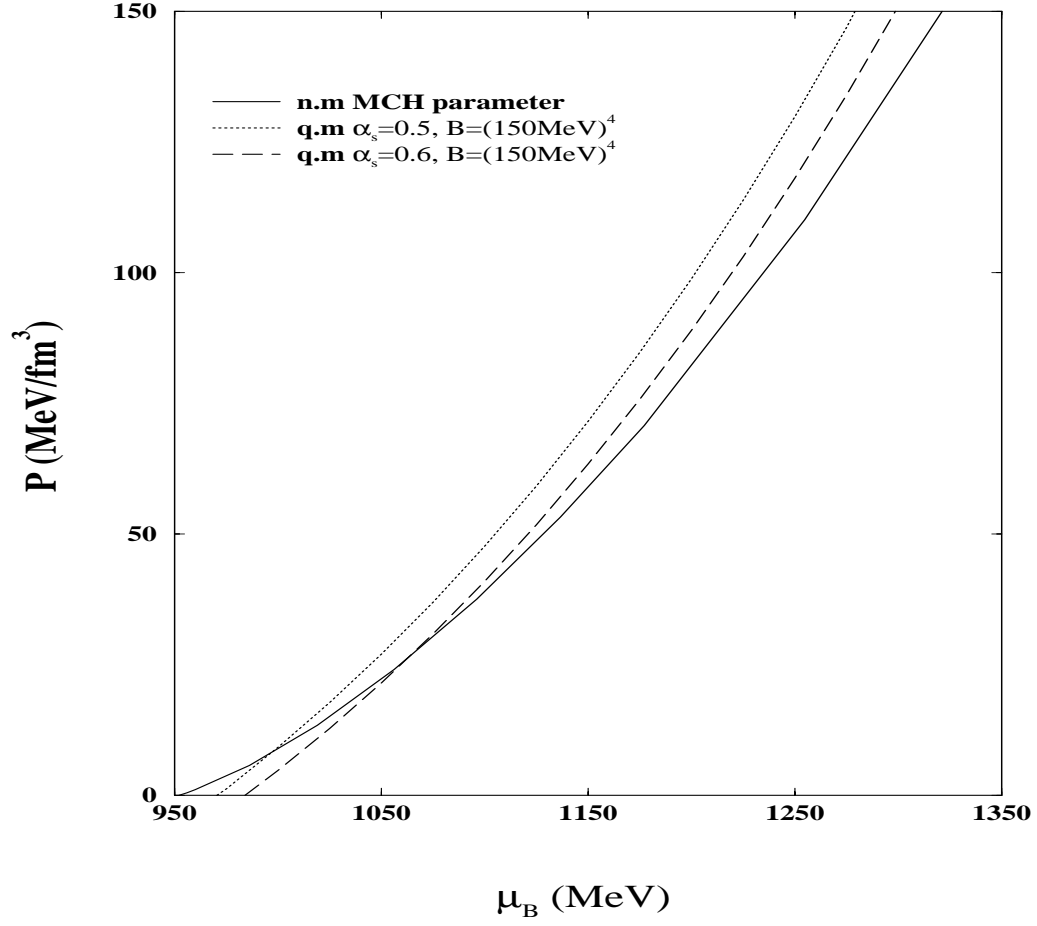


Figure 3: Pressure (P) vs. baryon chemical potential for nuclear matter and quark matter with various  $\alpha_s$  with different bag pressure (B)

## 4 Hybrid Stars.

Having established the existence of a phase transition, we now proceed to study the structure of a hybrid star. For the description of neutron star which can generate curvature in the space time geometry due to high concentration of matter, one has to apply Einstein's general theory of relativity. The space-time geometry generated by a spherical neutron star, described by the Schwarzschild metric can be represented in the form<sup>26</sup>

$$ds^2 = -e^{\nu(r)} dt^2 + [1 - 2M(r)/r]^{-1} dr^2 + r^2 [d\Theta^2 + \sin^2 \Theta d\phi^2]. \quad (13)$$

The Tolman-Oppenheimer-Volkoff (TOV) equations which determine the star structure and the geometry, in dimensionless forms<sup>26,27</sup>, are given by

$$\frac{d\hat{P}(\hat{r}r_0)}{d\hat{r}} = -\hat{G} \frac{[\hat{\epsilon}(\hat{r}r_0) + \hat{P}(\hat{r}r_0)][\hat{M}(\hat{r}r_0) + 4\pi a\hat{r}^3\hat{P}(\hat{r}r_0)]}{\hat{r}^2[1 - 2\hat{G}\hat{M}(\hat{r}r_0)/\hat{r}]}, \quad (14)$$

$$\hat{M}(\hat{r}r_0) = 4\pi a \int_0^{\hat{r}r_0} d\hat{r}' \hat{r}'^2 \hat{\epsilon}(\hat{r}'r_0), \quad (15)$$

and the metric function,  $\nu(r)$ , relating the element of time at  $r = \infty$  is given by<sup>26</sup>

$$\frac{d\nu(\hat{r}r_0)}{d\hat{r}} = 2\hat{G} \frac{[\hat{M}(\hat{r}r_0) + 4\pi a\hat{r}^3\hat{P}(\hat{r}r_0)]}{\hat{r}^2[1 - 2\hat{G}\hat{M}(\hat{r}r_0)/\hat{r}]} . \quad (16)$$

The following substitutions have been made in above Eqns.(14-16).

$$\hat{\epsilon} \equiv \epsilon/\epsilon_c, \quad \hat{P} \equiv P/\epsilon_c, \quad \hat{r} \equiv r/r_0, \quad \hat{M} \equiv M/M_\odot \quad (17)$$

Here ,

$$a \equiv \epsilon_c r_0^3/M_\odot, \quad \hat{G} \equiv \frac{GM_\odot}{f_1 r_0} \quad (18)$$

with  $f_1 = 197.327$  MeV fm and  $r_0 = 3 \times 10^{19}$  fm.

The quantities with hats are dimensionless in above equations. The gravitational constant  $G = 6.7079 \times 10^{-45}$  MeV<sup>-2</sup>.

For complete calculation of a stellar model, one has to integrate Eqns.(14-16) from the star's center at  $r = 0$  with a given central density  $\epsilon_c$  as input until the pressure  $P(r)$  at the surface vanishes. With any reasonable central energy density, we expect that at the center we shall have only quark matter. Hence we shall be using here the equation of state for quark matter through Eqn.(10) with  $\hat{P}(0) = P(\epsilon_c)$ . We then integrate the TOV equations until the pressure and density decrease to their critical values at  $r = r_c$ . For  $r > r_c$ , we shall have equation of state for  $\beta$ -stable nuclear matter where pressure will change continuously but the energy density will have a discontinuity at  $r = r_c$ . The TOV equations with nuclear matter equation of state are continued until the pressure vanishes which defines the surface of the star. This completes the calculations for stellar

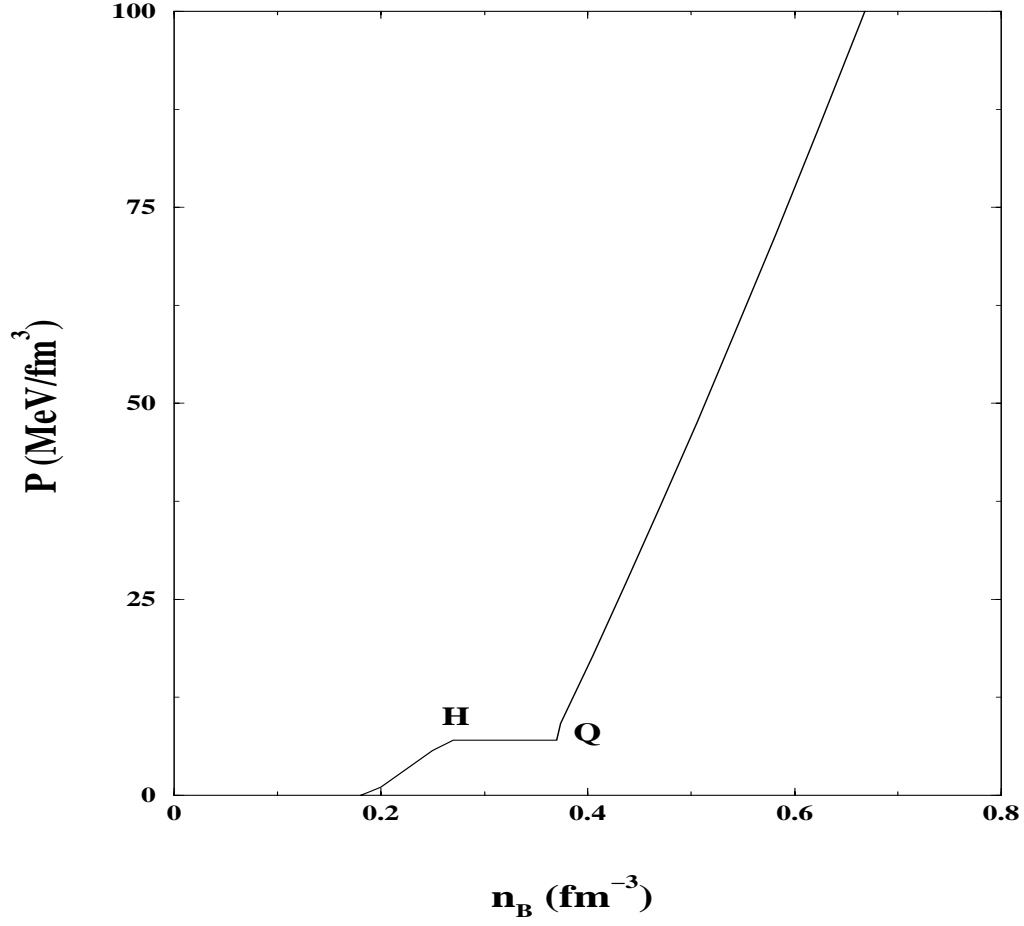


Figure 4: Pressure ( $P$ ) vs. number density( $n_B$ ) of hybrid star.

model for a hybrid neutron star, whose mass and radius can be calculated for different central densities.

Fig.[4] shows behavior of pressure versus number density in the vicinity of a first order phase transition in a system having one chemical potential corresponding to conserved baryon number<sup>8</sup>. Points labeled H and Q mark the end of the hadronic and beginning of the quark phase, the intervening region representing the mixed phase. The two equal pressure points at the opposite ends of the mixed phase are mapped onto the same radial point in the star. These aspects are also illustrated in Fig.[5].

Through out the calculation of hybrid star we have used  $\alpha_s = 0.5$  and  $B = (150\text{MeV})^4$  for quark sector. The energy density profile obtained from Eqns.(14-16) are plotted in

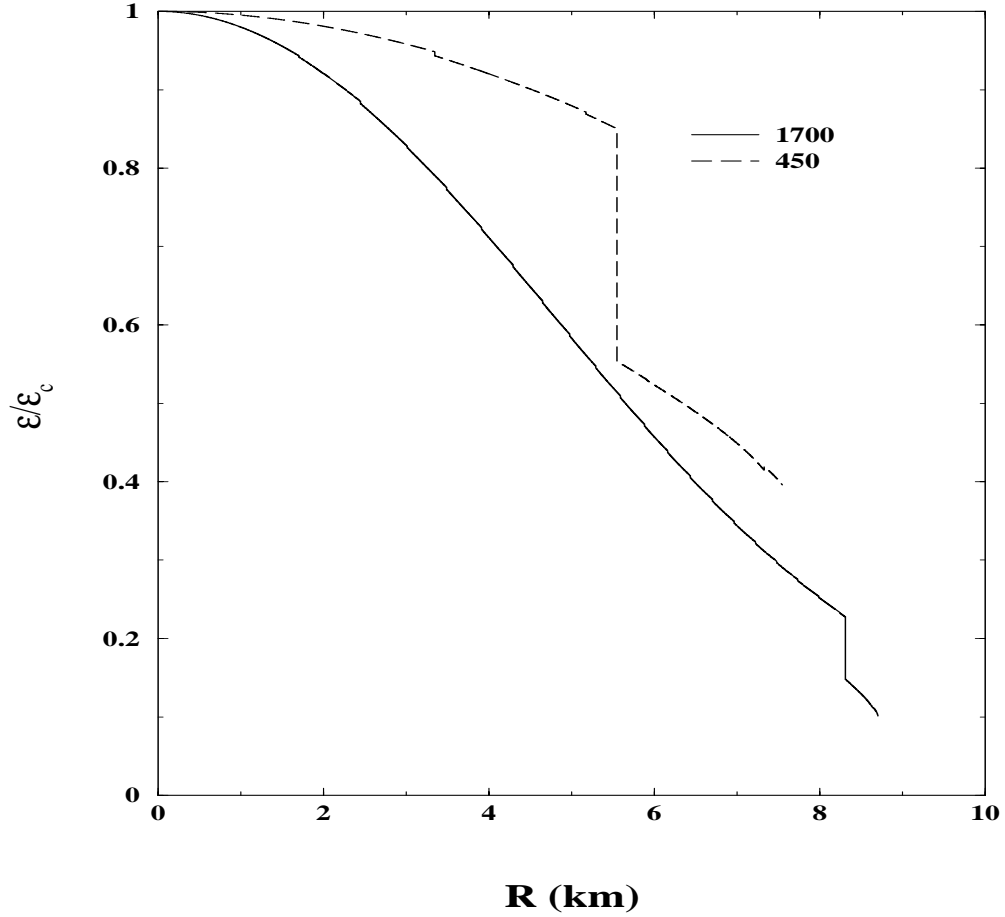


Figure 5: Energy density profile for central densities  $\epsilon_c = 1700 \text{ MeV/fm}^3$  (solid line) and  $\epsilon_c = 450 \text{ MeV/fm}^3$  (dashed line).

Fig.[5] for central energy density  $\epsilon_c = 450 \text{ MeV/fm}^3$  and  $\epsilon_c = 1700 \text{ MeV/fm}^3$ . For core energy densities greater than the critical energy density ( $\simeq 387 \text{ MeV/fm}^3$ ) the core consists of quark matter. As we go away from the core towards the surface through TOV equations, when the critical pressure is reached, the density drops discontinuously indicating a first order phase transition. Thus for central density of  $450 \text{ MeV/fm}^3$  such a star has a quark matter core of radius 5.5 kms with nuclear matter crust of about 2.2 kms, whereas for  $\epsilon_c = 1700 \text{ MeV/fm}^3$ , the quark matter core radius is 8.2 kms with nuclear matter crust of about 0.6 kms. Hence it is clear that if we take smaller central energy densities then nuclear matter is expected to be more abundant in a hybrid star.

We have plotted in Fig.[6] the mass of hybrid star as a function of central energy density to examine the stability of such a star. Hence we have two branches of solutions. Pure neutron star at lower densities  $\epsilon_c < \epsilon_{nm}^{cr}$  and hybrid stars at central densities  $\epsilon_c > \epsilon_{nm}^{cr}$ . Taking into account the stability of such stars under density fluctuations require  $dM/d\epsilon_c > 0$ <sup>26</sup>. As may be seen from the figure,  $dM/d\epsilon_c$  becomes negative around 1700 MeV/fm<sup>3</sup> after which it may collapse into black holes<sup>26,28</sup>. This yields the maximum mass of hybrid star as  $M \simeq 1.69 M_\odot$ . Fig.[7] shows the mass as a function of radius obtained for different central densities varying in the range of 450 MeV/fm<sup>3</sup> to 1700 MeV/fm<sup>3</sup> for such a star which indicates the maximum radius to be around 9.3 kms. To check the stability of our result against the errors in values of symmetric energy ( $32 \pm 6$  MeV) and the nuclear incompressibility ( $300 \pm 50$  MeV), we have also computed the coupling constants ( $C_\rho, C_\omega, C_\sigma, B$  and  $C$ ) for four cases of symmetric energy value 32 MeV with incompressibility 250 and 350 MeV and the incompressibility of 300 MeV with symmetric values 26 and 38 MeV. The maximum values for  $M$  and  $R$  are found to be  $M = 1.69_{-0.001}^{+0.005} M_\odot$  and  $R = 9.3_{-0.05}^{+0.21}$  kms. Such small effect of symmetric energy and incompressibility on maximum values of  $M$  and  $R$  can be attributed to the fact that only about 15% of the radius of the hybrid star contains nuclear matter.

We also calculate the surface gravitational redshift  $Z_s$  of photons which is given by<sup>29</sup>

$$Z_s = \frac{1}{\sqrt{1 - 2GM/R}} - 1 \quad (19)$$

In Fig.[8] we have plotted  $Z_s$  as a function of  $M/M_\odot$ . In this context it may be mentioned here that our result for the surface redshifts lie in the range of 0.2 to 0.5 as determined from gamma ray bursters<sup>30</sup>.

We then compute the relativistic Keplerian angular velocity  $\Omega_k$  given by<sup>31</sup>

$$\frac{\Omega_k}{10^4 \text{sec}^{-1}} = 0.72 \sqrt{\frac{M/M_\odot}{(R/10 \text{km})^3}} \quad (20)$$

as for neutron stars. Fig.[9] shows our result for variation of relativistic Keplerian angular velocity as a function of  $M/M_\odot$  for such a star. We find that the  $\Omega_k$  has an

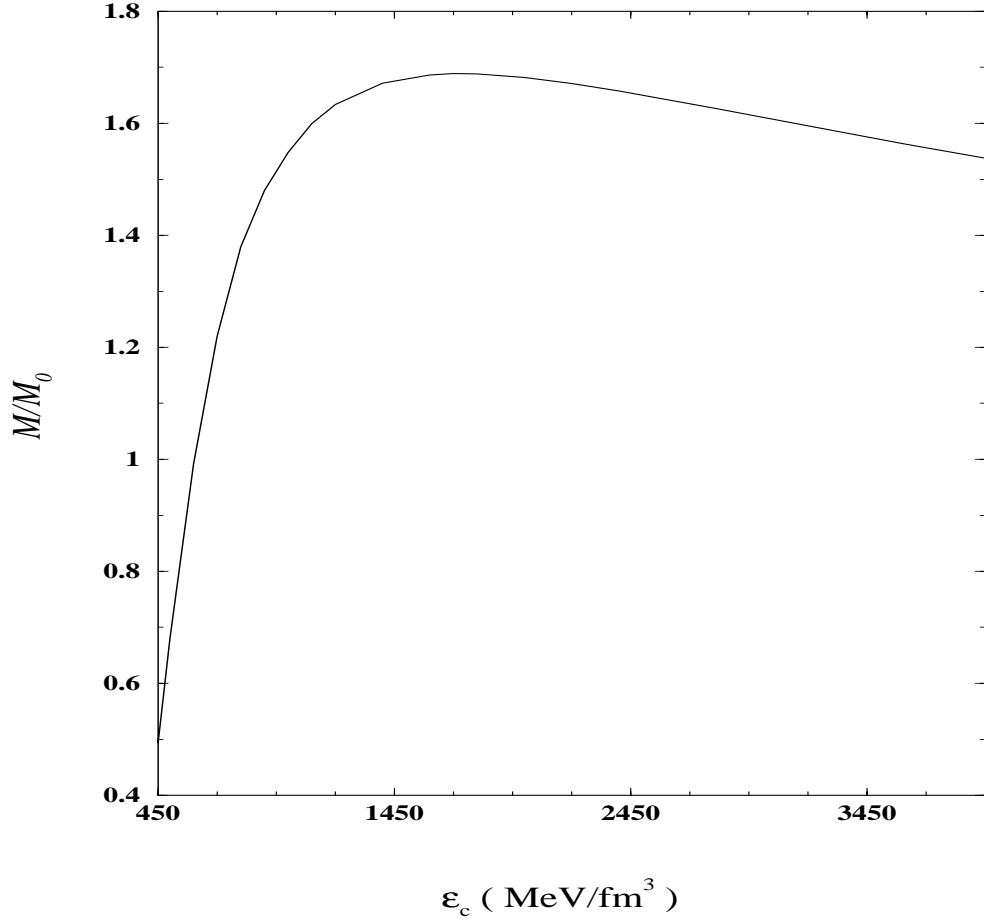


Figure 6: The mass( $M/M_\odot$ ) of the hybrid star as a function of central energy density( $\epsilon_c$ ).

inverse relationship after an initial increase with the mass of the star beyond  $1.69M_\odot$ . This indicates that we can not have mass of hybrid star more than about  $1.69M_\odot$ . We also observe that the maximum value of  $\Omega_k$  being near about  $10^4 \text{sec}^{-1}$  implies the time period to be less than 0.5 millisecond.

## 5 Conclusions

Within the formalism of a modified SU(2) chiral sigma model (Eqn.1) we found that a first order phase transition exists between the nuclear phase and quark phase at density of about two to three times the nuclear matter density. Quark matter equation of state

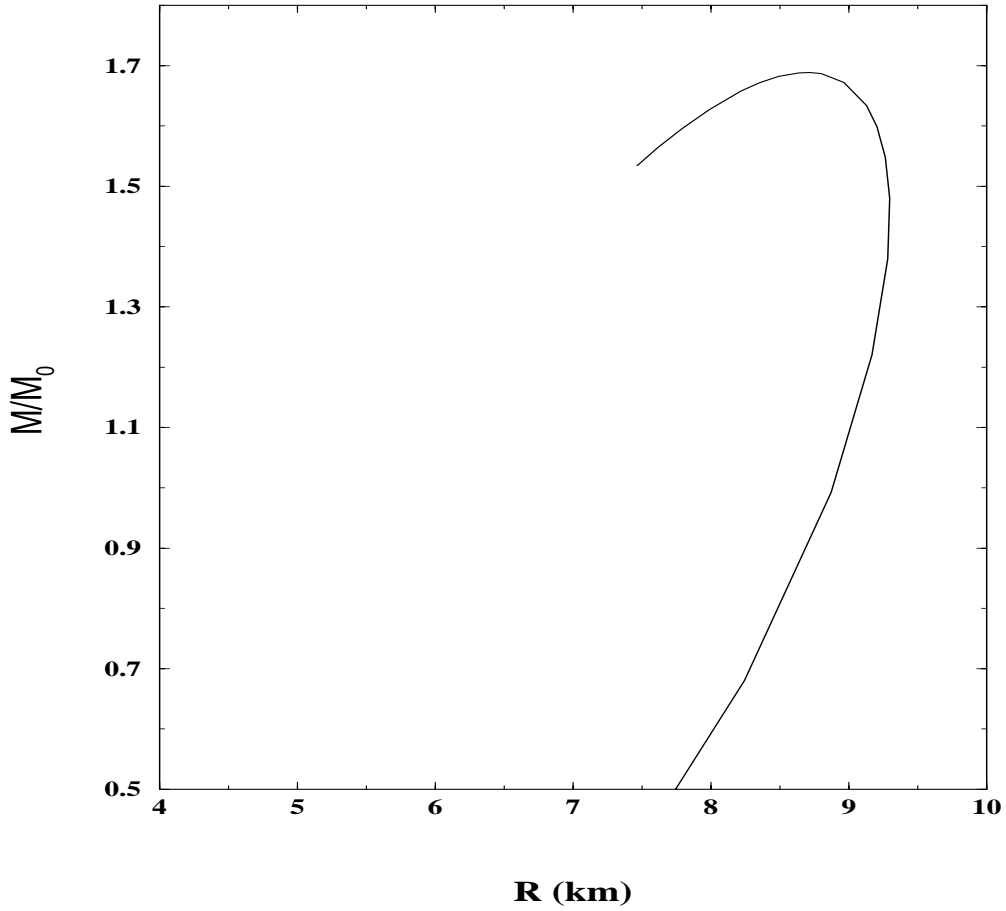


Figure 7: Mass as a function of radius for the hybrid star.

has the parameters:  $m_s$ ,  $\alpha_s$  and  $B$ . Our results show that the critical parameters while increasing with increasing value of  $\alpha_s$  and  $B$ , exhibit greater sensitivity with respect to  $\alpha_s$ .

The phase transition from nuclear matter to quark matter indicates that the core of a neutron star consists of quark matter. To obtain a stable hybrid star solution, we have solved TOV equations using appropriate equations of state with a given central energy density  $\epsilon_c$  and have taken  $\alpha_s = 0.5$  and  $B = (150 \text{ MeV})^4$  for quark matter EOS. It is observed that a stable hybrid star with a quark core and a nuclear matter crust exist upto  $\epsilon_c \simeq 1700 \text{ MeV/fm}^3$  beyond which instability is indicated. For  $\epsilon_c$  varying from  $450 \text{ MeV/fm}^3$  to  $1700 \text{ MeV/fm}^3$  the mass of hybrid star varies from  $0.5$  to  $1.69 M_\odot$  and the



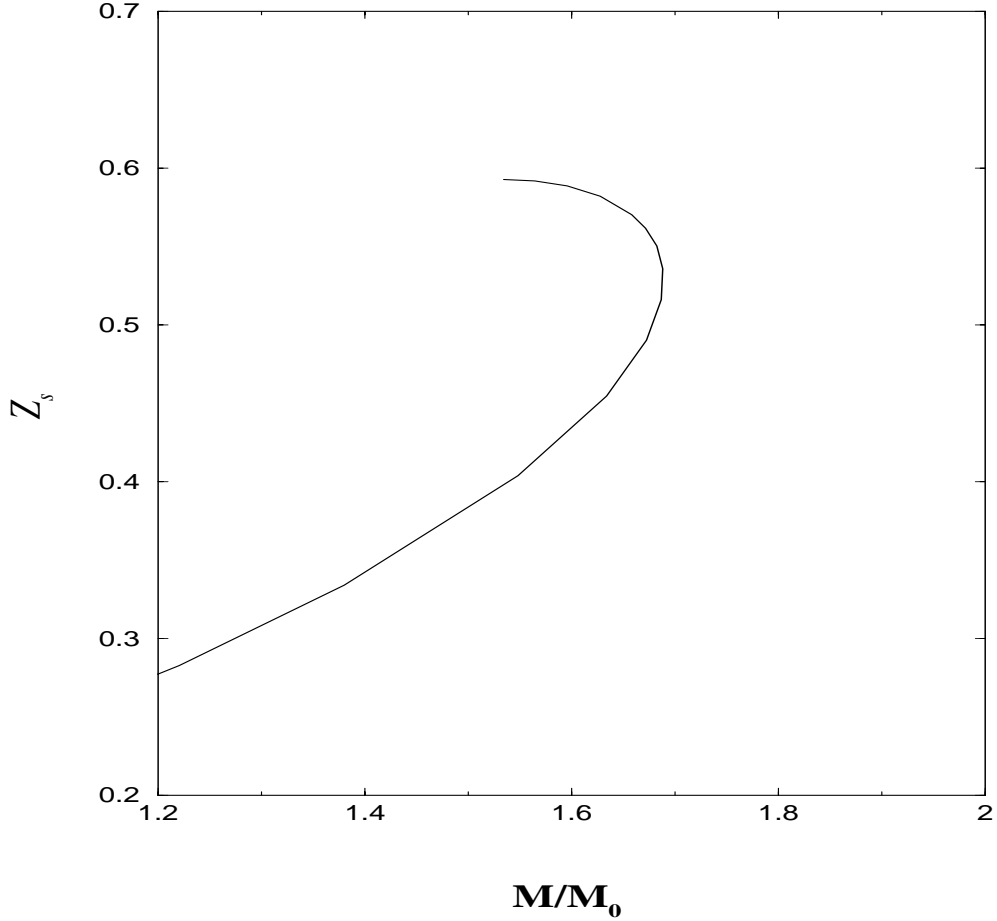


Figure 8: The surface gravitational redshift( $Z_s$ ) as a function of star mass( $M/M_\odot$ )

radius from 7.7 to 8.8 kms with a quark core of about 5 to 8 kms respectively. Our results, thus, indicate that the bulk of hybrid star is composed of quark matter with a crust of nuclear matter. We find that the maximum mass and radius of hybrid stars to be about  $1.69 M_\odot$  and 9.3 kms respectively depending on the values of the parameters used in the model. If we compare these mass and radius values of the hybrid stars with those obtained using this model<sup>10</sup> for pure neutron star (such as  $M = 2.1M_\odot$  and  $R = 12.1$  kms), we find that the hybrid stars are more compact than a normal neutron star. This result is in exact agreement with the general expectation since the EOS of quark matter forming the core of hybrid star is supposed to be softer than that of neutron matter because of QCD asymptotic freedom. The greater compactness of the star also

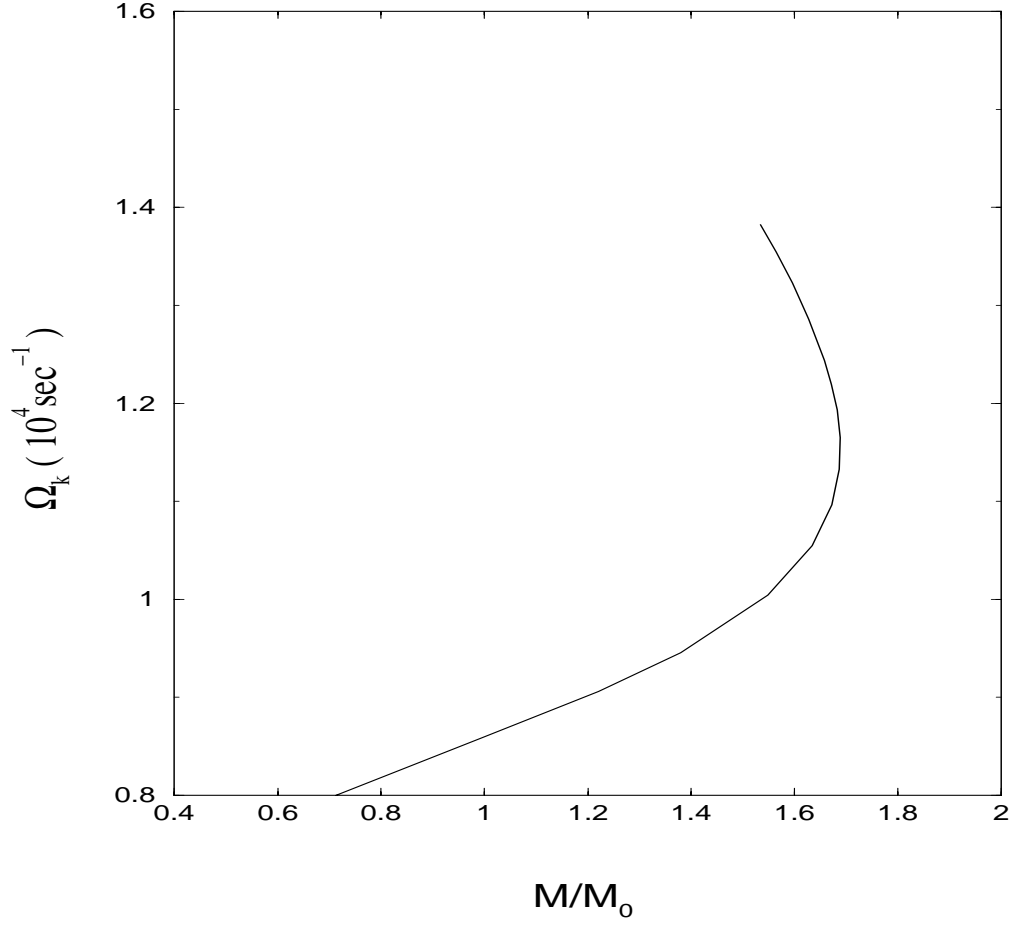


Figure 9: The Keplerian angular velocity( $\Omega_k$ ) as a function of star mass( $M/M_{\odot}$ )

leads to smaller time period lying in the submillisecond range as obtained by us. It is also observed that the surface gravitational redshift and relativistic Keplerian angular velocity of the hybrid stars can not increase beyond  $M/M_{\odot} = 1.69$  ; showing a decrease with increase in  $M/M_{\odot}$  beyond this value.

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